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Spontaneous supersymmetry breaking and instanton sum in 2D type IIA superstring theory

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Mainly based on

- T. Kuroki and F. S., Nucl. Phys. B **867** (2013) 448, arXiv 1208.3263
- T. Kuroki and F. S., JHEP **1403** (2014) 006, arXiv 1306.3561
- M. G. Endres, T. Kuroki, F. S. and H. Suzuki, Nucl. Phys. B **876** (2013) 758, arXiv 1308.3306
- S. M. Nishigaki and F. S., to be published in JHEP, arXiv 1405.1633

1 Introduction

◇ Solvable matrix models for 2D quantum gravity or noncritical string theories were vigorously investigated around 1990.

- as toy models for critical string theories, in particular focused on nonperturbative aspects.

- But, little has been known about (solvable) matrix models corresponding to noncritical superstrings with target-space SUSY.
  We would like to consider such matrix models.

- We hope our analysis helpful to analyze matrix models for critical superstrings.

In this Talk,

◇ I would like to discuss correspondence between

A simple zero-dimensional SUSY double-well matrix model (MM)

and

2D type IIA superstring on a nontrivial RR background.

An interesting example of MMs for superstrings with target-space SUSY,
in which various amplitudes (not protected by SUSY) are explicitly calculable.

◇ Full nonperturbative expression of the MM free energy is computed in its double scaling limit.

SUSY is spontaneously broken due to instantons.

$\Downarrow$

In the type IIA theory,

SUSY is dynamically broken by a nonperturbative effect.

Contents

2 SUSY double-well MM

[Kuroki-F.S. 2009]

$$S_{\rm MM} = N{\rm tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right],$$

where

B, ϕ: $N \times N$ hermitian matrices (Bosonic),

ψ, $\bar{\psi}$: $N \times N$ Grassmann-odd matrices (Fermionic).

- SUSY:

$$Q\phi = \psi, \quad Q\psi = 0, \quad Q\bar{\psi} = -iB, \quad QB = 0,$$
$$\bar{Q}\phi = -\bar{\psi}, \quad \bar{Q}\bar{\psi} = 0, \quad \bar{Q}\psi = -iB, \quad \bar{Q}B = 0.$$

$$\Rightarrow Q^2 = \bar{Q}^2 = \{Q, \bar{Q}\} = 0 \text{ (nilpotent)}$$

- B, ψ, $\bar{\psi}$ integrated out

$$S_{\rm MM} \to N{\rm tr}\,\frac{1}{2}(\phi^2 - \mu^2)^2 - \ln\det(\phi \otimes 1\!\!1_N + 1\!\!1_N \otimes \phi)$$

↑

Double-well scalar potential

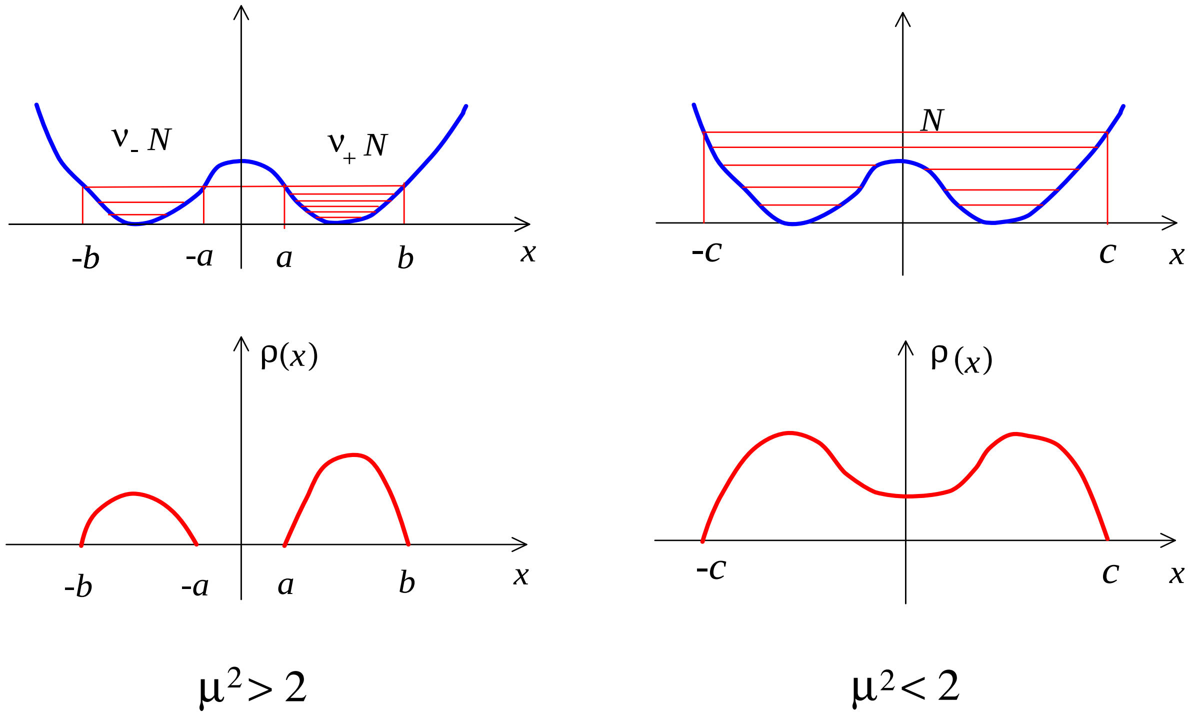

Figure 1: **(Left)**: "SUSY preserving" solution , **(Right)**: SUSY breaking solution at the planar limit.

◇ Large-N saddle point solution for $\rho(x) \equiv \frac{1}{N}\mathrm{tr}\,\delta(x-\phi)$: Planar limit

[Kuroki-F.S. 2010]

$\nu_\pm$: filling fractions $\nu_+ + \nu_- = 1$

- $\underline{\mu^2 > 2}$

  (large-N free energy) $= 0$, $\langle \frac{1}{N}\mathrm{tr}\, B^n \rangle = 0 \ (n = 1, 2, \cdots)$

  strongly suggest that SUSY is preserved.

  Note that $\mathrm{tr}\, B^n = Q\, \mathrm{tr}\, (i\bar{\psi} B^{n-1}) = \bar{Q}\, \mathrm{tr}(i\psi B^{n-1})$.

  $\Rightarrow$ The SUSY minima are infinitely degenerate, parametrized by (ν_+, ν_-).

- $\underline{\mu^2 < 2}$

  SUSY breaking one-cut solution:

  (large-N free energy) $\neq 0$, $\langle \frac{1}{N}\mathrm{tr}\, B \rangle \neq 0$

- 3rd order phase transition between these two phases.

  The 3rd derivative of the free energy w.r.t. μ^2 has a jump.

3 Planar correlation functions

In the SUSY phase ($\mu^2 > 2$),

$$\left\langle \frac{1}{N}\,\mathrm{tr}\,\phi^n \right\rangle_0 = \int dx\, x^n \rho(x)$$
$$= (\nu_+ + (-1)^n \nu_-)\,(2+\mu^2)^{n/2}\, F\left(-\frac{n}{2}, \frac{3}{2}, 3; \frac{4}{2+\mu^2}\right)$$

- reduces to a polynomial of μ^2 for n even:

$$\left\langle \frac{1}{N}\mathrm{tr}\,\phi^2 \right\rangle_0 = \mu^2, \qquad \left\langle \frac{1}{N}\mathrm{tr}\,\phi^4 \right\rangle_0 = 1 + \mu^4, \qquad \cdots .$$

- exhibits logarithmic singular behavior as $\mu^2 \to 2$ for n odd:

$$\omega \equiv \tfrac{1}{4}(\mu^2 - 2)$$

$$\left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k+1} \right\rangle_0 = (\nu_+ - \nu_-)\left[(\text{const.})\,\omega^{k+2}\ln\omega + (\text{less singluar})\right].$$

- We also computed planar two- and three-point functions for

$$\Phi_{2k+1} \sim \frac{1}{N}\mathrm{tr}\,\phi^{2k+1}.$$

The results so far suggest

$$\langle \Phi_{2k_1+1} \cdots \Phi_{2k_n+1} \rangle_{C,0} = (\nu_+ - \nu_-)^n (\text{const.})\, \omega^{3+\sum_{i=1}^{n}(k_i-1)} (\ln \omega)^n + (\text{less singular})$$

- For fermions ($\Psi_{2k+1} \sim \frac{1}{N}\mathrm{tr}\,\psi^{2k+1}$, $\bar{\Psi}_{2k+1} \sim \frac{1}{N}\mathrm{tr}\,\bar{\psi}^{2k+1}$),

$$\left\langle \Psi_{2k+1} \bar{\Psi}_{2\ell+1} \right\rangle_{C,0} = \delta_{k,\ell}\,(\text{const.})\,(\nu_+ - \nu_-)^{2k+1} \omega^{2k+1} \ln \omega + (\text{less singular})$$

Logarithmic behavior reminds us of 2D string theory.
But, the higher powers are new. ($\leftarrow$ RR background)

4 2D type IIA superstring [Kutasov-Seiberg 1990, Ita-Nieder-Oz 2005]

- (Target space) $= (x, \varphi) \sim$ Cylinder,
  where $x \in S^1$ with self-dual radius $(R = 1)$ and φ: Liouville.

- Holomorphic EM tensor (except ghost part) on string worldsheet:

$$T_m = -\frac{1}{2}(\partial x)^2 - \frac{1}{2}\psi_x \partial \psi_x - \frac{1}{2}(\partial \varphi)^2 + \frac{Q}{2}\partial^2 \varphi - \frac{1}{2}\psi_\ell \partial \psi_\ell$$

  with $Q = 2$.

- Target-space SUSY is nilpotent.

$$q_+(z) = e^{-\frac{1}{2}\phi - \frac{i}{2}H - ix}(z), \qquad Q_+ = \oint \frac{dz}{2\pi i}\, q_+(z),$$

$$\bar{q}_-(\bar{z}) = e^{-\frac{1}{2}\bar{\phi} + \frac{i}{2}\bar{H} + i\bar{x}}(\bar{z}), \qquad \bar{Q}_- = \oint \frac{d\bar{z}}{2\pi i}\, \bar{q}_-(\bar{z}),$$

  where $\psi_\ell \pm i\psi_x = \sqrt{2}e^{\mp iH}$.

  $\Rightarrow Q_+^2 = \bar{Q}_-^2 = \{Q_+, \bar{Q}_-\} = 0$. ($\leftarrow$ Same as the matrix model!)

- Vertex operators (holomorphic sector):

$$\text{NS sector } (-1)\text{-picture}: \quad T_k(z) = e^{-\phi + ikx + p_\ell \varphi}(z)$$

$$\text{R sector } (-\tfrac{1}{2})\text{-picture}: \quad V_{k,\,\epsilon}(z) = e^{-\frac{1}{2}\phi + \frac{i}{2}\epsilon H + ikx + p_\ell \varphi}(z)$$

with helicity $\epsilon = \pm 1$.

Phyiscal vertex operators (Winding background): [Ita-Nieder-Oz 2005]

(NS, NS) : $T_k(z)\,\bar{T}_{-k}(\bar{z})$ $(k \in \mathbf{Z} + \frac{1}{2})$ "tachyon" winding

(R+, R−) : $V_{k,\,+1}(z)\,\bar{V}_{-k,\,-1}(\bar{z})$ $(k = \frac{1}{2}, \frac{3}{2}, \cdots)$

(R−, R+) : $V_{-k,\,-1}(z)\,\bar{V}_{k,\,+1}(\bar{z})$ $(k = 0, 1, 2, \cdots)$

RR 2-form field strength winding

(NS, R−) : $T_{-k}(z)\,\bar{V}_{-k,\,-1}(\bar{z})$ $(k = \frac{1}{2}, \frac{3}{2}, \cdots)$ fermion(−) momentum

(R+, NS) : $V_{k,\,+1}(z)\,\bar{T}_k(\bar{z})$ $(k = \frac{1}{2}, \frac{3}{2}, \cdots)$ fermion(+) momentum

They represent massless particles with $p_\ell = 1 - |k|$.

5 Correspondence between the MM and the IIA theory

Under the identification of supercharges between the MM and the type IIA theory:

$$(Q, \bar{Q}) \Leftrightarrow (Q_+, \bar{Q}_-).$$

$\Rightarrow$ SUSY transformation properties lead to

$$\Phi_1 = \frac{1}{N}\mathrm{tr}\,\phi \Leftrightarrow \int d^2z\, V_{\frac{1}{2},+1}(z)\,\bar{V}_{-\frac{1}{2},-1}(\bar{z}) \qquad (\mathrm{R}+,\ \mathrm{R}-),$$

$$\Psi_1 = \frac{1}{N}\mathrm{tr}\,\psi \Leftrightarrow \int d^2z\, T_{-\frac{1}{2}}(z)\,\bar{V}_{-\frac{1}{2},-1}(\bar{z}) \qquad (\mathrm{NS},\ \mathrm{R}-),$$

$$\bar{\Psi}_1 = \frac{1}{N}\mathrm{tr}\,\bar{\psi} \Leftrightarrow \int d^2z\, V_{\frac{1}{2},+1}(z)\,\bar{T}_{\frac{1}{2}}(\bar{z}) \qquad (\mathrm{R}+,\ \mathrm{NS}),$$

$$\frac{1}{N}\mathrm{tr}(-iB) \Leftrightarrow \int d^2z\, T_{-\frac{1}{2}}(z)\,\bar{T}_{\frac{1}{2}}(\bar{z}) \qquad (\mathrm{NS},\ \mathrm{NS}).$$

Quartet w.r.t. $(Q, \bar{Q})$ $\Leftrightarrow$ Quartet w.r.t. $(Q_+, \bar{Q}_-)$

Furthermore, it is natural to extend it to higher $k(=1,2,\cdots)$ as

$$\Phi_{2k+1} = \frac{1}{N}\mathrm{tr}\,\phi^{2k+1} + \cdots \Leftrightarrow \int d^2z\, V_{k+\frac{1}{2},+1}(z)\,\bar{V}_{-k-\frac{1}{2},-1}(\bar{z}),$$

$$\Psi_{2k+1} = \frac{1}{N}\mathrm{tr}\,\psi^{2k+1} + \cdots \Leftrightarrow \int d^2z\, T_{-k-\frac{1}{2}}(z)\,\bar{V}_{-k-\frac{1}{2},-1}(\bar{z}),$$

$$\bar{\Psi}_{2k+1} = \frac{1}{N}\mathrm{tr}\,\bar{\psi}^{2k+1} + \cdots \Leftrightarrow \int d^2z\, V_{k+\frac{1}{2},+1}(z)\,\bar{T}_{k+\frac{1}{2}}(\bar{z}),$$

(Single trace operators in the MM) $\Leftrightarrow$ (Integrated vertex operators in IIA)

(Powers of matrices) $\Leftrightarrow$ (Windings or Momenta)

Note:

- RR 2-form field strength in (R−, R+) is a singlet under the target-space SUSYs Q_+, $\bar{Q}_-$, and appears to have no MM counterpart.

- Expectation values of operators measuring the RR charge (e.g. $\langle \Phi_{2k+1} \rangle_0$) are nonvanishing in the MM.

$\Rightarrow$ The MM is considered to correspond to IIA on a background of the RR 2-form. $\nu_+ - \nu_- \Leftrightarrow$ (RR flux)

We can explicitly check the correspondence by computing various amplitudes in the IIA theory. [Kuroki-F.S. 2014]

Note: Correlation functions in the IIA strings on (R−, R+) background:

$$\left\langle\!\left\langle \prod_i \mathcal{V}_i \right\rangle\!\right\rangle \equiv \left\langle \left(\prod_i \mathcal{V}_i\right) e^{W_{\rm RR}} \right\rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \left\langle \left(\prod_i \mathcal{V}_i\right) (W_{\rm RR})^n \right\rangle,$$

where $W_{\rm RR}$ is invariant under the target-space SUSYs:

$$W_{\rm RR} = (\nu_+ - \nu_-) \sum_{k\in Z} a_k\, \omega^{k+1} \mathcal{V}_k^{\rm RR}, \qquad (a_k : \text{numerical consts.})$$

$$\mathcal{V}_k^{\rm RR} \equiv \begin{cases} \int d^2z\, V_{k,\,-1}(z) \bar{V}_{-k,\,+1}(\bar{z}) & (p_\ell = 1 - |k|,\ k = 0, -1, -2, \cdots) \\ \int d^2z\, V^{(\rm nonlocal)}_{-k,\,-1}(z) \bar{V}^{(\rm nonlocal)}_{k,\,+1}(\bar{z}) & (p_\ell = 1 + |k|,\ k = 1, 2, \cdots). \end{cases}$$

- We consider the case of $(\nu_+ - \nu_-)$ small, and the RR background is treated as exponentiated vertex operators:

- Higher powers of $\ln \omega$ comes from resonances among external particles and the (R−,R+) background. $|\ln \omega| \Leftrightarrow$ (Liouville volume)

6 Nonperturbative SUSY breaking in the MM

◇ SUSY double-well MM

$$S_{\rm MM} = N{\rm tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right].$$

After integrating out matrices other than ϕ, the partition function is expressed in terms of eigenvalues λ_i $(i = 1, \cdots, N)$ as

$$\begin{aligned} Z_{\rm MM} &= \tilde{C}_N \int \left(\prod_{i=1}^{N} d\lambda_i\right) \triangle(\lambda)^2 \prod_{i,j=1}^{N} (\lambda_i + \lambda_j)\, e^{-N\sum_{i=1}^{N}\frac{1}{2}(\lambda_i^2-\mu^2)^2} \\ &= \sum_{\nu_- N=0}^{N} \frac{N!}{(\nu_+ N)!(\nu_- N)!}\, Z_{(\nu_+,\nu_-)}, \end{aligned}$$

where the partition function in the (ν_+, ν_-) sector is defined by the integration region

$$\int_0^\infty \prod_{i=1}^{\nu_+ N} d\lambda_i \qquad \int_{-\infty}^0 \prod_{j=\nu_+ N+1}^{N} d\lambda_j.$$

By $\lambda_j \to -\lambda_j$ $(j = \nu_+ N + 1, \cdots, N)$, it is easy to see

$$Z_{(\nu_+,\nu_-)} = (-1)^{\nu_- N}\, Z_{(1,0)}.$$

Thus, the total partition function vanishes:

$$Z_{\mathrm{MM}} = \sum_{\nu_- N=0}^{N} \frac{N!}{(\nu_+ N)!(\nu_- N)!}\, Z_{(\nu_+,\nu_-)} = (1 + (-1))^N\, Z_{(1,0)} = 0.$$

$\Rightarrow$ Expectation values normalized by Z_{MM} become ill-defined.

Let us regularize it as

$$Z_\alpha \equiv \sum_{\nu_- N=0}^{N} \frac{N!}{(\nu_+ N)!(\nu_- N)!} e^{-i\alpha\nu_- N} Z_{(\nu_+,\nu_-)} = (1 - e^{-i\alpha})^N Z_{(1,0)}.$$

◇ Order parameter of spontaneous SUSY breaking:

$$\left\langle \frac{1}{N}\mathrm{tr}(iB) \right\rangle_\alpha = \frac{1}{N^2}\frac{1}{Z_\alpha}\frac{\partial}{\partial(\mu^2)} Z_\alpha = \frac{1}{N^2}\frac{1}{Z_{(1,0)}}\frac{\partial}{\partial(\mu^2)} Z_{(1,0)}$$

is independent of α and well-defined in the limit $\alpha \to 0$.

Problem reduces to computing $Z_{(1,0)}$.

After the variable change $x_i = \mu^2 - \lambda_i^2$ (Nicolai mapping),

$$Z_{(1,0)} = \tilde{C}_N \int_{-\infty}^{\mu^2} \left(\prod_{i=1}^{N} dx_i \right) \triangle(x)^2 \, e^{-N \sum_{i=1}^{N} \frac{1}{2} x_i^2},$$

◇ Techniques in the random matrix theory [Tracy-Widom 1994] give a closed form for the partition function in the double scaling limit

$$N \to \infty, \qquad \mu^2 \to 2 \qquad \text{with} \qquad s = N^{2/3}(\mu^2 - 2) \quad \text{fixed}$$

as

$$F = -\ln Z_{(1,0)} = \int_s^\infty (x - s) q(x)^2 dx,$$

where $q(x)$ is a solution to the Painléve II differential equation

$$q''(x) = xq(x) + 2q(x)^3$$

with $q(x) \sim \mathrm{Ai}(x)$ as $x \to +\infty$.

- The solution is unique. [Hastings-McLeod 1980]
- $g_{st} \sim 1/N \sim s^{-3/2}$

  $\Rightarrow$ $s \gg 1$: weakly coupled, $0 < s \ll 1$: strongly coupled.

6.1 Weak coupling expansion

◇ By using the Airy kernel [Tracy-Widom 1994]

$$K_{\rm Ai}(s,t) \equiv \frac{{\rm Ai}(s){\rm Ai}'(t) - {\rm Ai}'(s){\rm Ai}(t)}{s-t},$$

the free energy expressed as an instanton sum

$$F = -\ln Z_{(1,0)} = \sum_{k=1}^{\infty} F_{k-{\rm inst.}}$$

is expanded as

$$\begin{aligned} F_{k-{\rm inst.}} &= \frac{1}{k}\int_s^{\infty} dt_1 \ldots dt_k \, K_{\rm Ai}(t_1,t_2)\, K_{\rm Ai}(t_2,t_3) \cdots K_{\rm Ai}(t_k,t_1) \\ &\sim \frac{1}{k}\left(\frac{1}{16\pi s^{3/2}}\, e^{-\frac{4}{3}s^{3/2}}\right)^k \left[1 + a_1^{(k)} s^{-3/2} + a_2^{(k)} s^{-3} + \cdots\right]. \end{aligned}$$

- $N^{4/3} \cdot \left\langle \frac{1}{N}\mathrm{tr}(iB) \right\rangle^{(1,0)} = -\frac{dF}{ds} \neq 0$

  $\uparrow$

  Wave-function renormalization

  $\Rightarrow$ SUSY is spontaneously broken due to instantons.

- Nambu-Goldstone fermions: $\frac{1}{N}\mathrm{tr}\,\bar{\psi}$ ($\leftarrow$ breaking of Q)

  $\frac{1}{N}\mathrm{tr}\,\psi$ ($\leftarrow$ breaking of $\bar{Q}$)

- The Airy-kernel expression of $F_{k-\mathrm{inst.}}$ contains all perturbative contributions around the k-instanton configuration.

6.2 Strong coupling expansion

◇ The Taylor series expansion of $F = \int_s^\infty (x-s)q(x)^2 dx$ around $s = 0$ is

$$F = 0.0311059853 - 0.0690913807s + 0.0673670913s^2 - 0.0361399144s^3 + \cdots$$

This gives strong coupling expansion of the IIA superstring theory.

- The strongly coupled limit is regular!
- The expression of F is smoothly continued to the $s < 0$ region.

  ($\Leftrightarrow \mu^2 < 2$)

  The 3rd order phase transition in the planar limit becomes smooth crossover in the double scaling limit.

  Singular behavior at the "string tree level" is smeared by quantum effects.

  Similar to the unitary one-matrix model.

The result implies

"nonSUSY string" $\Leftarrow$ (smooth crossover) $\Rightarrow$ 2DIIA superstring

$(s < 0)$ $\qquad\qquad\qquad\qquad$ $(s > 0)$

$F_{\text{pert.}} \neq 0$ $\qquad\qquad\qquad\qquad$ $F_{\text{pert.}} = 0, \; F_{\text{nonpert.}} \neq 0$

holds at least concerning the partition function.

7 Summary and discussions

◇ We computed correlation functions in the double-well SUSY MM, and discussed its correspondence to 2D type IIA superstring theory on (R−,R+) background by computing amplitudes in both sides.

- Case of $(\nu_+ - \nu_-)$ not small?
  Related to black-hole (cigar) target space? cf. [Hori-Kapustin 2001]

- Massive excitations ("discrete states")
  $\mathrm{tr}(\phi^k\psi^\ell\phi^m\psi^n\cdots) \Leftrightarrow$ (polynomial of $\partial x,\ \partial\varphi, \cdots)e^{ikx+p_\ell\varphi+\cdots}$
  are suggested by SUSY transformation properties.

- MMs for higher-dimensional noncritical superstrings $(D=4,6,8,(10))$?
  $D = \quad 2 \quad + \quad (D-2)$ [Kutasov-Seiberg 1990]
  ↗ ↖
  (x,φ): Nilpotent SUSY $\qquad$ $\mathbf{R}^{D-2}$: Usual SUSY generating translations

◇ The full nonperturbative expression of the free energy of the MM is obtained.

- Strong coupling expansion
  $\Rightarrow$ existence of the S-dual theory (noncritical M theory)?
- D-brane computation in the type IIA side.
- Connection between nonSUSY string and 2DIIA superstring.

Thank you very much for your attention!