Spontaneous supersymmetry breaking and instanton sum in 2D type IIA superstring theory*

Fumihiko Sugino†
Okayama Institute for Quantum Physics,
Kyoyama 1-9-1, Kita-ku, Okayama 700-0015, Japan

Abstract

We consider a double-well supersymmetric matrix model and its interpretation as a nonperturbative definition of two-dimensional type IIA superstring theory. The interpretation is confirmed by direct comparison of symmetries and amplitudes in both sides of the matrix model and the IIA superstring theory. Next, we obtain the full nonperturbative free energy of the matrix model in terms of the Tracy-Widom distribution in random matrix theory. Its weak coupling expansion implies spontaneous supersymmetry breaking due to instantons, and strong coupling behavior suggests the existence of a well-defined S-dual theory. Furthermore, from the expression of the free energy, we see a smooth connection between a non-supersymmetric string theory and the IIA superstring theory.

PACS codes: 02.10.Yn; 11.25.Pm; 11.25.Sq

1. Introduction

Solvable matrix models for two-dimensional quantum gravity or noncritical string theory had been vigorously investigated around 1990, focusing on nonperturbative aspects in string theory [5]. While this approach has been successful for bosonic string theory, little has been known for superstring theory, in particular theories possessing target-space supersymmetry (SUSY). So it would be worth while to considering (solvable) matrix models describing superstring theory with target-space SUSY. In this article, we discuss correspondence between a simple zero-dimensional SUSY double-well matrix model and two-dimensional type IIA superstring theory on a nontrivial Ramond-Ramond (RR) background. Then, nonperturbative effect of the matrix model is computed in its double scaling limit. As a result, we find that SUSY is spontaneously broken due to instantons in the matrix

* This is mainly based on the work with Tsunehide Kuroki, Michael G. Endres, Hiroshi Suzuki and Shinsuke M. Nishigaki [1, 2, 3, 4]. The work is supported in part by a Grant-in-Aid for Scientific Research (C), 25400289.
† e-mail address: fusugino@gmail.com

model. According to the correspondence, this suggests spontaneous SUSY breaking at the nonperturbative level in the type IIA superstring theory.

We hope our analysis is helpful to understand nonperturbative dynamics of matrix models of super Yang-Mills type for critical superstring theory [6, 7, 8].

2. Review of double-well SUSY matrix model

We give a brief review of a SUSY matrix model given by the following action [9]:

$$S = N\mathrm{tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right], \tag{1}$$

where B and ϕ are $N \times N$ hermitian matrices, and ψ and $\bar{\psi}$ are $N \times N$ Grassmann-odd matrices. The action is invariant under SUSY transformations generated by Q and $\bar{Q}$:

$$Q\phi = \psi, \quad Q\psi = 0, \quad Q\bar{\psi} = -iB, \quad QB = 0, \tag{2}$$
$$\bar{Q}\phi = -\bar{\psi}, \quad \bar{Q}\bar{\psi} = 0, \quad \bar{Q}\psi = -iB, \quad \bar{Q}B = 0, \tag{3}$$

which leads to the nilpotency: $Q^2 = \bar{Q}^2 = \{Q, \bar{Q}\} = 0$. A large-$N$ saddle point for the eigenvalue distribution of the matrix ϕ: $\rho(x) \equiv \frac{1}{N}\mathrm{tr}\,\delta(x - \phi)$ has two supports $[-b, -a] \cup [a, b]$ in the case of $\mu^2 > 2$ (the two-cut solution) and a single support $[-c, c]$ for $\mu^2 < 2$ (the one-cut solution). Here, $a = \sqrt{\mu^2 - 2}$, $b = \sqrt{\mu^2 + 2}$ and $c = \sqrt{\frac{2}{3}}\left(\mu^2 + \sqrt{\mu^4 + 12}\right)^{1/2}$. The two-cut solution is characterized by the filling fractions (ν_+, ν_-) satisfying $\nu_+ + \nu_- = 1$, which indicate that the ratio of eigenvalues distributing over $[a, b]$ and those distributing over $[-b, -a]$ is $\nu_+ : \nu_-$. The large-N free energy and the expectation values $\left\langle \frac{1}{N}\mathrm{tr}\,B^n \right\rangle$ $(n = 1, 2, \cdots)$ evaluated at the two-cut solution turn out to all vanish [9], strongly suggesting that the solution preserves SUSY. Thus, we conclude that the SUSY minima are infinitely degenerate and parametrized by (ν_+, ν_-) in the simple large-N limit (the planar limit). On the other hand, the one-cut solution gives nonzero values of $\left\langle \frac{1}{N}\mathrm{tr}B \right\rangle$ and of the large-N free energy, showing that SUSY is broken [10]. The transition between the SUSY phase ($\mu^2 > 2$) and the SUSY broken phase ($\mu^2 < 2$) is of the third order. Namely, the third derivative of the free energy with respect to μ^2 has a jump at $\mu^2 = 2$.

In the SUSY phase ($\mu^2 > 2$) with the filling fraction (ν_+, ν_-), the planar correlation functions of the operators Φ_{2k+1} $(k = 0, 1, 2, \cdots)$ [1]:

$$\Phi_{2k+1} \equiv \frac{1}{N}\mathrm{tr}\,\phi^{2k+1} + (\text{mixing}) \tag{4}$$

[1] The second term "(mixing)" in (4) consists of lower even powers of ϕ. Its explicit form is given in ref. [1].

behave as [2]

$$\left\langle \prod_{i=1}^{n} \Phi_{2k_i+1} \right\rangle_{C,0} \sim (\nu_+ - \nu_-)^n \, u_{k_1,\cdots,k_n} \, \omega^{3+\sum_{i=1}^{n}(k_i-1)} \, (\ln \omega)^n \tag{5}$$

with $\omega \equiv \frac{1}{4}(\mu^2 - 2)$ and the constants $u_{k_1,\cdots,k_n}$ taking the form as

$$\begin{aligned} u_k &= \frac{2^{k+2}}{\pi} \frac{(2k+1)!!}{(k+2)!}, \\ u_{k,\ell} &= -\frac{1}{2\pi^2} \frac{1}{k+\ell+1} \frac{(2k+1)!}{(k!)^2} \frac{(2\ell+1)!}{(\ell!)^2}, \\ &\cdots. \end{aligned} \tag{6}$$

The planar two-point functions of fermions $\Psi_{2k+1} = \frac{1}{N}\mathrm{tr}\,\psi^{2k+1}+$(mixing) and $\bar{\Psi}^{2\ell+1} = \frac{1}{N}\mathrm{tr}\,\bar{\psi}^{2\ell+1}$ + (mixing) are evaluated as

$$\left\langle \Psi_{2k+1} \bar{\Psi}_{2\ell+1} \right\rangle_{C,0} \sim \delta_{k,\ell} \, v_k \, (\nu_+ - \nu_-)^{2k+1} \omega^{2k+1} \ln \omega \tag{7}$$

with v_k being constants $v_0 = \frac{1}{\pi}$, $v_1 = \frac{6}{\pi}$, $\cdots$.

3. 2D type IIA superstring

The type II superstring theory discussed in refs. [11, 12, 13] has the target space $(\varphi, x) \in$ (Liouville direction) $\times$ (S^1 with self-dual radius). The holomorphic energy-momentum tensor on the string world-sheet is

$$T = -\frac{1}{2}(\partial x)^2 - \frac{1}{2}\psi_x \partial \psi_x - \frac{1}{2}(\partial \varphi)^2 + \partial^2 \varphi - \frac{1}{2}\psi_\ell \partial \psi_\ell \tag{8}$$

excluding ghosts' part. ψ_x and ψ_ℓ are superpartners of x and φ, respectively. Target-space supercurrents in the type IIA theory

$$q_+(z) = e^{-\frac{1}{2}\phi(z) - \frac{i}{2}H(z) - ix(z)}, \qquad \bar{q}_-(\bar{z}) = e^{-\frac{1}{2}\bar{\phi}(\bar{z}) + \frac{i}{2}\bar{H}(\bar{z}) + i\bar{x}(\bar{z})} \tag{9}$$

exist only on the S^1 target space of the self-dual radius. ϕ ($\bar{\phi}$) is the holomorphic (anti-holomorphic) bosonized superconformal ghost, and the fermions are bosonized as $\psi_\ell \pm i\psi_x = \sqrt{2}\, e^{\mp iH}$, $\bar{\psi}_\ell \pm i\bar{\psi}_x = \sqrt{2}\, e^{\mp i\bar{H}}$. In addition, we should care about cocycle factors in order to realize the anticommuting nature between q_+ and $\bar{q}_-$. Supercurrents with the cocycle factors are

$$\hat{q}_+(z) = e^{\pi\beta(\frac{1}{2}p_{\bar{\phi}} - i\frac{1}{2}p_{\bar{h}} - ip_{\bar{x}})} \, q_+(z), \qquad \hat{\bar{q}}_-(\bar{w}) = e^{-\pi\beta(\frac{1}{2}p_\phi + i\frac{1}{2}p_h + ip_x)} \, \bar{q}_-(\bar{w}), \tag{10}$$

[2]The suffix "C" means that connected parts are taken, and the symbol "$\sim$" denotes equality up to additive less singular terms.

where $\beta \in \mathbf{Z} + \frac{1}{2}$, and p_ϕ, p_h and p_x ($p_{\bar{\phi}}$, $p_{\bar{h}}$ and $p_{\bar{x}}$) are momentum modes of holomorphic part (anti-holomorphic part) of free bosons [2]. Then the supercharges

$$\hat{Q}_+ = \oint \frac{dz}{2\pi i}\, \hat{q}_+(z), \qquad \hat{\bar{Q}}_- = \oint \frac{d\bar{z}}{2\pi i}\, \hat{\bar{q}}_-(\bar{z}) \tag{11}$$

are nilpotent $\hat{Q}_+^2 = \hat{\bar{Q}}_-^2 = \{\hat{Q}_+, \hat{\bar{Q}}_-\} = 0$, which matches the property of the supercharges Q and $\bar{Q}$ in the matrix model.

The spectrum except special massive states is represented by the NS "tachyon" [3] vertex operator (in (-1) picture):

$$T_k = e^{-\phi + ikx + p_\ell \varphi}, \qquad \bar{T}_{\bar{k}} = e^{-\bar{\phi} + i\bar{k}\bar{x} + p_\ell \bar{\varphi}}, \tag{12}$$

and by the R vertex operator (in $(-\frac{1}{2})$ picture):

$$V_{k,\epsilon} = e^{-\frac{1}{2}\phi + \frac{i}{2}\epsilon H + ikx + p_\ell \varphi}, \qquad \bar{V}_{\bar{k},\bar{\epsilon}} = e^{-\frac{1}{2}\bar{\phi} + \frac{i}{2}\bar{\epsilon}\bar{H} + i\bar{k}\bar{x} + p_\ell \bar{\varphi}} \tag{13}$$

with $\epsilon, \bar{\epsilon} = \pm 1$. Cocycle factors for vertex operators are introduced as [2]

$$\begin{aligned}
&\hat{T}_k(z) = e^{\pi\beta(p_{\bar{\phi}} + ikp_{\bar{x}})}\, T_k(z), \quad \hat{\bar{T}}_{\bar{k}}(\bar{z}) = e^{-\pi\beta(p_\phi + i\bar{k}p_x)}\, \bar{T}_{\bar{k}}(\bar{z}), \\
&\hat{V}_{k,\epsilon}(z) = e^{\pi\beta(\frac{1}{2}p_{\bar{\phi}} + i\frac{\epsilon}{2}p_{\bar{h}} + ikp_{\bar{x}})}\, V_{k,\epsilon}(z), \\
&\hat{\bar{V}}_{\bar{k},\bar{\epsilon}}(\bar{z}) = e^{-\pi\beta(\frac{1}{2}p_\phi + i\frac{\bar{\epsilon}}{2}p_h + i\bar{k}p_x)}\, \bar{V}_{\bar{k},\bar{\epsilon}}(\bar{z}).
\end{aligned} \tag{14}$$

Locality with the supercurrents, mutual locality, superconformal invariance and the level matching condition determine physical vertex operators. As discussed in [13], there are two consistent sets of physical vertex operators - "momentum background" and "winding background". Here, we consider the "winding background". [4] The physical spectrum in the "winding background" is given by

$$\begin{aligned}
&(\text{NS, NS}): && \hat{T}_k\, \hat{\bar{T}}_{-k} && (k \in \mathbf{Z} + \tfrac{1}{2}), \\
&(\text{R+, R}-): && \hat{V}_{k,+1}\, \hat{\bar{V}}_{-k,-1} && (k = \tfrac{1}{2}, \tfrac{3}{2}, \cdots), \\
&(\text{R}-, \text{R+}): && \hat{V}_{-k,-1}\, \hat{\bar{V}}_{k,+1} && (k = 0, 1, 2, \cdots), \\
&(\text{NS, R}-): && \hat{T}_{-k}\, \hat{\bar{V}}_{-k,-1} && (k = \tfrac{1}{2}, \tfrac{3}{2}, \cdots), \\
&(\text{R+, NS}): && \hat{V}_{k,+1}\, \hat{\bar{T}}_k && (k = \tfrac{1}{2}, \tfrac{3}{2}, \cdots),
\end{aligned} \tag{15}$$

[3] In two dimensions, "tachyon" is not truely tachyonic but massless.

[4] We can repeat the parallel argument for "momentum background" in the type IIB theory, which is equivalent to the "winding background" in the type IIA theory through T-duality with respect to the S^1 direction.

where we take a branch of $p_\ell = 1 - |k|$ satisfying the locality bound $p_\ell \leq Q/2 = 1$ [14].

Comparing the SUSY transformation property of the matrix-model operators and that of the above vertex operators leads to the correspondence [1]:

$$\begin{aligned}
\Phi_{2k+1} \quad &\Leftrightarrow \quad \mathcal{V}_\phi(k) \equiv \int d^2z\, \hat{V}_{k+\frac{1}{2},+1}(z)\, \hat{\tilde{V}}_{-k-\frac{1}{2},-1}(\bar{z}), \\
\Psi_{2k+1} \quad &\Leftrightarrow \quad \mathcal{V}_\psi(k) \equiv \int d^2z\, \hat{T}_{-k-\frac{1}{2}}(z)\, \hat{\tilde{V}}_{-k-\frac{1}{2},-1}(\bar{z}), \\
\bar{\Psi}_{2k+1} \quad &\Leftrightarrow \quad \mathcal{V}_{\bar{\psi}}(k) \equiv \int d^2z\, \hat{V}_{k+\frac{1}{2},+1}(z)\, \hat{\tilde{T}}_{k+\frac{1}{2}}(\bar{z}), \\
\frac{1}{N}\mathrm{tr}\,(-iB) \quad &\Leftrightarrow \quad \mathcal{V}_B(0) \equiv \int d^2z\, \hat{T}_{-\frac{1}{2}}(z)\, \hat{\tilde{T}}_{\frac{1}{2}}(\bar{z})
\end{aligned} \tag{16}$$

for $k = 0, 1, 2, \cdots$. Note that (R$-$, R$+$) operators are singlets under the target-space SUSYs $\hat{Q}_+, \hat{\tilde{Q}}_-$, and appear to have no counterpart in the matrix model side. Since the expectation value of operators measuring an RR charge $\langle \Phi_{2k+1} \rangle_0$ does not vanish as seen in (5), we conjecture that the matrix model corresponds to the type IIA theory on a nontrivial background of the (R$-$, R$+$) fields. As a check of the conjecture, introducing the (R$-$, R$+$) background in the form of vertex operators

$$\begin{aligned}
W_{\mathrm{RR}} \quad &= \quad (\nu_+ - \nu_-) \sum_{k \in \mathbf{Z}} a_k\, \mu_1^{k+1} \mathcal{V}_k^{\mathrm{RR}}, \\
\mathcal{V}_k^{\mathrm{RR}} \quad &\equiv \quad \begin{cases} \int d^2z\, \hat{V}_{k,-1}(z) \hat{\tilde{V}}_{-k,+1}(\bar{z}) & (p_\ell = 1 - |k|,\ k \leq 0) \\ \int d^2z\, \hat{V}^{(\mathrm{nonlocal})}_{-k,-1}(z) \hat{\tilde{V}}^{(\mathrm{nonlocal})}_{k,+1}(\bar{z}) & (p_\ell = 1 + |k|,\ k \geq 1) \end{cases}
\end{aligned} \tag{17}$$

with a_k being numerical constants, we consider correlation functions among integrated vertex operators $\mathcal{V}_i$ in the IIA theory on a nontrivial (R$-$, R$+$) background in the form:

$$\left\langle\!\!\left\langle \prod_i \mathcal{V}_i \right\rangle\!\!\right\rangle \equiv \left\langle \left(\prod_i \mathcal{V}_i \right) e^{W_{\mathrm{RR}}} \right\rangle. \tag{18}$$

When the strength of the background $(\nu_+ - \nu_-)$ is small, we can compute the correlation functions by expanding the background as $e^{W_{\mathrm{RR}}} = 1 + W_{\mathrm{RR}} + \frac{1}{2!}(W_{\mathrm{RR}})^2 + \cdots$. Results of the computation support the equivalence of the matrix model to the IIA superstring theory on the (R$-$, R$+$) background [2]. Interestingly, in the calculation of the IIA amplitudes, we see that the multiple powers of $\ln \omega$ in the matrix-model correlators (5) are identified with resonances among the external particles and the (R$-$, R$+$) background.

4. Nonperturbative SUSY breaking in the matrix model

In this section, we obtain the full nonperturbative free energy of the matrix model as the Tracy-Widom distribution in random matrix theory in the double scaling limit

$$N \to \infty, \quad \omega \to 0 \quad \text{with } s \equiv 4N^{2/3}\omega \text{ fixed.} \tag{19}$$

In its weakly coupled region (s: large), instanton effects can be seen in the matrix model which are nonperturbative in $1/N$. Although such effects are of the order e^{-N} and vanish in the simple large N limit, we will see that they are nonvanishing in the double scaling limit (19).

The partition function of the matrix model given by the action (1) is expressed as

$$\begin{aligned} Z &= \int d^{N^2}\phi\, e^{-N\frac{1}{2}\mathrm{tr}(\phi^2-\mu^2)^2}\ \det(\phi\otimes\mathbf{1}+\mathbf{1}\otimes\phi) \\ &= \tilde{C}_N \int \Big(\prod_{i=1}^N d\lambda_i\Big) \triangle(\lambda)^2 \prod_{i,j=1}^N (\lambda_i+\lambda_j)\, e^{-N\sum_{i=1}^N \frac{1}{2}(\lambda_i^2-\mu^2)^2}, \end{aligned} \tag{20}$$

after integrating out matrices other than ϕ. Here, $\mathbf{1}$ is an $N\times N$ unit matrix, λ_i ($i=1,\cdots,N$) are eigenvalues of ϕ, and $\triangle(\lambda)$ denotes the Vandermonde determinant $\triangle(\lambda)=\prod_{i>j}(\lambda_i-\lambda_j)$. $\tilde{C}_N$ is an numerical factor depending only on N given by

$$\frac{1}{\tilde{C}_N} = \int \Big(\prod_{i=1}^N d\lambda_i\Big) \triangle(\lambda)^2\, e^{-N\sum_{i=1}^N \frac{1}{2}\lambda_i^2} = (2\pi)^{\frac{N}{2}} \frac{\prod_{k=0}^N k!}{N^{\frac{N^2}{2}}}. \tag{21}$$

Contributions to the partition function are divided by sectors labeled by the filling fraction (ν_+,ν_-) as

$$Z = \sum_{\nu_- N=0}^{N} \frac{N!}{(\nu_+N)!(\nu_-N)!}\, Z_{(\nu_+,\nu_-)} \tag{22}$$

with

$$\begin{aligned} Z_{(\nu_+,\nu_-)} \equiv\ & \tilde{C}_N \int_0^\infty \left(\prod_{i=1}^{\nu_+N} d\lambda_i\right) \int_{-\infty}^0 \left(\prod_{j=\nu_+N+1}^{N} d\lambda_j\right) \left(\prod_{n=1}^N 2\lambda_n\right) \\ & \times \left\{\prod_{n>m}(\lambda_n^2-\lambda_m^2)^2\right\} e^{-N\sum_{i=1}^N \frac{1}{2}(\lambda_i^2-\mu^2)^2}. \end{aligned} \tag{23}$$

Here, it is easy to see

$$Z_{(\nu_+,\nu_-)} = (-1)^{\nu_-N} Z_{(1,0)}, \tag{24}$$

which leads to the vanishing partition function:

$$Z = (1 + (-1))^N Z_{(1,0)} = 0. \tag{25}$$

In order for expectation values normalized by the partition function to be well-defined, we regularize the partition function by introducing a factor $e^{-i\alpha\nu_- N}$ with small α in front of $Z_{(\nu_+,\nu_-)}$. The regularized partition function reads

$$Z_\alpha \equiv \sum_{\nu_- N=0}^{N} \frac{N!}{(\nu_+ N)!(\nu_- N)!}\, e^{-i\alpha\nu_- N} Z_{(\nu_+,\nu_-)} = (1 - e^{-i\alpha})^N\, Z_{(1,0)}. \tag{26}$$

Notice that calculations in perturbation theory of $1/N$ in section 2. concern the partition function in a single sector $(Z_{(\nu_+,\nu_-)})$, in which such a regularization was not needed. On the other hand, since nonperturbative contributions to be computed here possibly communicate among various sectors of filling fractions, we should consider the total partition function (22) and its null result requires the regularization.

The expectation value of $\frac{1}{N}\mathrm{tr}(iB)$ under the regularization (26) is expressed as

$$\left\langle \frac{1}{N}\mathrm{tr}\,(iB)\right\rangle_\alpha = \frac{1}{N^2}\frac{1}{Z_\alpha}\frac{\partial}{\partial(\mu^2)}Z_\alpha = \frac{1}{N^2}\frac{1}{Z_{(1,0)}}\frac{\partial}{\partial(\mu^2)}Z_{(1,0)} \tag{27}$$

due to a cancellation of the factor $(1-e^{-i\alpha})^N$ in (26) between the numerator and the denominator. The regularized expectation value $\left\langle \frac{1}{N}\mathrm{tr}\,(iB)\right\rangle_\alpha$ is independent of α and well-defined in the limit $\alpha \to 0$, and thus serves as an order parameter for spontaneous SUSY breaking.

4.1. Tracy-Widom distribution

Under the change of variables $x_i = -\lambda_i^2 + \mu^2$, the partition function $Z_{(1,0)}$ defined in (23) reduces to Gaussian matrix integrals

$$Z_{(1,0)} = \tilde{C}_N \int_{-\infty}^{\mu^2} \left(\prod_{i=1}^{N} dx_i\right) \triangle(x)^2\, e^{-N\sum_{i=1}^{N}\frac{1}{2}x_i^2}. \tag{28}$$

It seems almost trivial, but a nontrivial effect arises from the upper bound of the integration region. Techniques in random matrix theory [15] give a closed form for the partition function in the double scaling limit (19):

$$F(s) = -\ln Z_{(1,0)} = \int_s^\infty (x-s)q(x)^2 dx, \tag{29}$$

where $q(x)$ satisfies a Painlevé II differential equation

$$q''(x) = xq(x) + 2q(x)^3 \tag{30}$$

with the boundary condition

$$q(x) \to \mathrm{Ai}(x) \qquad (x \to +\infty). \tag{31}$$

Such a solution is unique and known as the Hastings-McLeod solution [16]. Since eq. (19) indicates that the string coupling constant $g_s \sim 1/N$ is proportional to $s^{-3/2}$, the region of $s \gg 1$ ($0 < s \ll 1$) describes the weakly (strongly) coupled IIA strings.

4.2. Weak coupling expansion

The partition function is given by the Fredholm determinant of the Airy kernel [15]

$$Z_{(1,0)} = \mathrm{Det}(1 - \hat{K}_{\mathrm{Ai}}|_{[s,\infty)}), \tag{32}$$

where the operator $\hat{K}_{\mathrm{Ai}}|_{[s,\infty)}$ can be represented as the integration kernel on the interval $[s,\infty)$:

$$K_{\mathrm{Ai}}(x,y) \equiv \frac{\mathrm{Ai}(x)\mathrm{Ai}'(y) - \mathrm{Ai}'(x)\mathrm{Ai}(y)}{x-y}. \tag{33}$$

From the above fact, it turns out that the weak coupling expansion (large-s expansion) of the free energy is expressed as an instanton sum [4]

$$F = -\ln Z_{(1,0)} = \sum_{k=1}^{\infty} F_{k-\mathrm{inst.}} \tag{34}$$

with

$$\begin{aligned} F_{k-\mathrm{inst.}} &= \frac{1}{k}\int_s^\infty dt_1 \dots dt_k\, K_{\mathrm{Ai}}(t_1,t_2)\, K_{\mathrm{Ai}}(t_2,t_3)\cdots K_{\mathrm{Ai}}(t_k,t_1) \\ &\sim \frac{1}{k}\left(\frac{1}{16\pi s^{3/2}}\, e^{-\frac{4}{3}s^{3/2}}\right)^k \left[1 + a_1^{(k)} s^{-3/2} + a_2^{(k)} s^{-3} + \cdots\right]. \end{aligned} \tag{35}$$

Some of the coefficients are

$$\begin{aligned} &a_1^{(1)} = -\frac{35}{24}, \quad a_2^{(1)} = \frac{3745}{1152}, \quad a_3^{(1)} = -\frac{805805}{82944}, \cdots \\ &a_1^{(2)} = -\frac{35}{12}, \quad a_2^{(2)} = \frac{619}{72}, \quad a_3^{(2)} = -\frac{592117}{20736}, \cdots \\ &a_1^{(3)} = -\frac{35}{8}, \quad a_2^{(3)} = \frac{2059}{128}, \quad a_3^{(3)} = -\frac{184591}{3072}, \cdots \\ &a_1^{(4)} = -\frac{35}{6}, \quad a_2^{(4)} = \frac{3701}{144}, \quad a_3^{(4)} = -\frac{1112077}{10368}, \cdots . \end{aligned} \tag{36}$$

The contribution to the free energy has no perturbative part and starts from nonperturbative effects of the instanton action $\frac{4}{3}s^{3/2} \propto N$ and its fluctuations expanded by $s^{-3/2} \propto N^{-1}$. It seems plausible that the non-perturbative contributions are provided by D-brane like objects. The order parameter of the SUSY breaking (with the wave function renormalization factor $N^{4/3}$), $N^{4/3}\left\langle \frac{1}{N}\mathrm{tr}(iB)\right\rangle^{(1,0)} = -F'(s)$, remains nonzero, implying that the target-space SUSY in the two-dimensional IIA theory is spontaneously broken by D-brane like objects. Corresponding Nambu-Goldstone fermions are identified with $\frac{1}{N}\mathrm{tr}\bar{\psi}$ and $\frac{1}{N}\mathrm{tr}\psi$ associated with the breaking of Q and $\bar{Q}$, respectively [3].

4.3. Strong coupling expansion

The Taylor series expansion of (29) around $s = 0$ is

$$\begin{aligned} F(s) &= 0.0311059853 - 0.0690913807s + 0.0673670913s^2 \\ &\quad -0.0361399144s^3 + \cdots, \end{aligned} \tag{37}$$

which gives strong coupling expansion of the IIA superstring theory. The strong coupling limit is regular and finite. In particular, the expression is smooth around $s = 0$ and there is no obstruction to be continued to the $s < 0$ region (i.e. $\mu^2 < 2$), whereas in section 2. we had seen the third order phase transition across the point $\mu^2 = 2$ in the planar limit. Thus, the singularity in the planar limit becomes smeared out in the double scaling limit. [5] In the string-theory perspective, singular behavior at the string tree level is smoothed out by quantum effects. Similar phenomenon can be seen in the unitary one-matrix model [17].

In the region of $s < 0$, the free energy has a perturbative series in $(-s)^{-3} \propto N^{-2}$, which seems to allow an interpretation as non-SUSY (type 0) string theory. Thus, we can see that the matrix model describes both of the IIA superstrings and type 0 strings in a unified manner at least concerning the free energy.

5. Summary and Discussion

We have computed planar correlation functions in the double-well SUSY matrix model, and discussed its correspondence to two-dimensional type IIA superstring theory on (R−,R+) background. This is an interesting example of matrix models for superstrings with target-space SUSY, which allows explicit calculation of various amplitudes of observables not restricted to those protected by SUSY.

It is interesting to examine the correspondence at deeper level, for instance in higher genus or higher point amplitudes and in amplitudes containing special massive operators. It is also important to discuss the correspondence in the off-shell formulation such as the hybrid formalism [18].

[5] From a physical viewpoint, it is considered that the free energy in the $\nu_+ = \nu_- = 1/2$ sector for $\mu^2 > 2$ smoothly connects to the free energy for $\mu^2 < 2$ in the double scaling limit. Due to (24), the former is essentially equal to (29) except an unimportant additive term.

Next, we have explicitly presented the full nonperturbative expression of the matrix-model free energy in a closed form. The result in the weakly coupled regime shows that the SUSY is spontaneously broken by nonperturbative effects due to instantons. In particular, the instanton effects survive in the double scaling limit, which implies that SUSY breaking takes place by nonperturbative dynamics in the target space of the type IIA superstring theory. It is interesting to reproduce the instanton contributions from dynamics of D-branes in the type IIA theory.

The free energy in the strongly coupled limit is smooth, which means that the singular behavior of the third order phase transition in the planar limit (at the string tree level) becomes smeared out in the double scaling limit (including quantum effects in the IIA strings). The regularity at $s = 0$ might suggest the existence of an S-dual theory, and the region $s < 0$ seems to describe a non-SUSY (type 0) string theory. It would be intriguing to identify such S-dual theory and to understand the moduli space of noncritical string theories given by the matrix model.

Acknowledgments

The author would like to thank the organizers of the 8th Mathematical Physics Meeting, especially Professors Branko Dragovich, Zoran Rakic and Igor Salom, for the invitation to the wonderful meeting and for warm hospitality.

References

[1] T. Kuroki and F. Sugino, *Nucl. Phys.* **B 867** (2013) 448.
[2] T. Kuroki and F. Sugino, *J. High Energy Phys.* **1403** (2014) 006.
[3] M. G. Endres, T. Kuroki, F. Sugino and H. Suzuki, *Nucl. Phys.* **B 876** (2013) 758.
[4] S. M. Nishigaki and F. Sugino, *J. High Energy Phys.* **1409** (2014) 104.
[5] For a review, see P. Di Francesco, P. H. Ginsparg and J. Zinn-Justin, *Phys. Rept.* **254** (1995) 1.
[6] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, *Phys. Rev.* **D 55** (1997) 5112.
[7] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, *Nucl. Phys.* **B 498** (1997) 467.
[8] R. Dijkgraaf, E. P. Verlinde and H. L. Verlinde, *Nucl. Phys.* **B 500** (1997) 43.
[9] T. Kuroki and F. Sugino, *Nucl. Phys.* **B 830** (2010) 434.
[10] T. Kuroki and F. Sugino, *Nucl. Phys.* **B 844** (2011) 409.
[11] D. Kutasov and N. Seiberg, *Phys. Lett.* **B 251** (1990) 67.
[12] S. Murthy, *J. High Energy Phys.* **0311** (2003) 056.
[13] H. Ita, H. Nieder and Y. Oz, *J. High Energy Phys.* **0506** (2005) 055.
[14] N. Seiberg, *Prog. Theor. Phys. Suppl.* **102** (1990) 319.
[15] C. A. Tracy and H. Widom, *Commun. Math. Phys.* **159** (1984) 151.
[16] S. P. Hastings and J. B. McLeod, *Arch. Rat. Mech. Anal.* **73** (1980) 31.
[17] V. Periwal and D. Shevitz, *Phys. Rev. Lett.* **64** (1990) 1326.
[18] P. A. Grassi and Y. Oz, *Preprint* arXiv:hep-th/0507168