Polarization of Photons in Matter-Antimatter Universe*

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Abstract

In this paper we demonstrate the possibility of generation of linear polarization of the electromagnetic field (EMF) due to the quantum effects of the EMF in matter-antimatter annihilation for anisotropic space of the I type according to Bianchi. The importance of the EMF polarisation concerning the physics of matter-antimatter annihilation in the early Universe relies on the fact that scalar fluctuations and the photon quantum effects can produce only linear polarisation and no circular polarisation. A measurement of the circular polarisation could therefore be interpreted as the detection of gravitational waves caused by matter-antimatter annihilation. Here we consider the generation of the polarization of the EMF in matter-antimatter annihilation process under idealized supposition that the medium is transparent and there is only the CMB radiation before matter-antimatter annihilation. The result of the paper is the assertion that the quantum effects of the EMF in the external gravitational field in the space of the I type according to Bianchi give contribution to the degree of polarization of the EMF in the quadrupole harmonics.

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1. Polarization of Photons in Matter-Antimatter Annihilation

1.1. Formalism of the EMF polarisation

A useful way to characterise the polarisation properties of the EMF is to use the Stokes parameters formalism [1, 2]. For a nearly monochromatic plane electro-magnetic wave propagating in the z direction,

$$E_x = a_x(t)\cos\left[\omega_0 t - \theta_x(t)\right], E_y = a_y(t)\cos\left[\omega_0 t - \theta_y(t)\right] \tag{1}$$

the Stokes parameters are defined by:

$$\begin{gathered} I \equiv \langle a_x^2\rangle + \langle a_y^2\rangle, \quad Q \equiv \langle a_x^2\rangle - \langle a_y^2\rangle, \\ U \equiv \langle 2a_x a_y \cos(\theta_x - \theta_y)\rangle, \quad V \equiv \langle 2a_x a_y \sin(\theta_x - \theta_y)\rangle, \end{gathered} \tag{2}$$

where the brackets $\langle\rangle$ represent time averages. The parameter I is simply the average intensity of the radiation. The polarisation properties are described by the remaining parameters: Q and U describe linear polarisation, while V describes circular polarisation. Unpolarised radiation (or natural light) is characterised by having $Q = U = V = 0$. The EMF polarisation is produced through the photon quantum effects (see below) which cannot generate circular polarisation. Then, we can write $V = 0$ always for.
The Stokes parameters Q and U are not scalar quantities. If we rotate the reference frame by an angle ϕ around the direction of observation, Q and U transform as:

$$Q' = Q\cos(2\phi) + U\sin(2\phi), U' = U\cos(2\phi) - Q\sin(2\phi). \tag{3}$$

We can define a *polarisation vector* $\mathbf{P}$ having:

$$|\mathbf{P}| = \left(Q^2 + U^2\right)^{1/2}, \qquad \alpha = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right). \tag{4}$$

Although $\mathbf{P}$ is a good way to visualise polarisation, it is not properly a vector, since it remains identical after a rotation by π around z, thus defining an orientation but not a direction. Mathematically, Q and U can be thought as the components of the second-rank symmetric trace-free tensor:

$$\mathbf{P}_{ab} = \frac{1}{2}\begin{pmatrix} Q & -U\sin\theta \\ -U\sin\theta & -Q\sin^2\theta \end{pmatrix}, \tag{5}$$

where the trigonometric functions come from having adopted a spherical coordinate system.

1.2. Formalism of the EMF radiation in anisotropic space of the I type according to Bianchi model

Let us consider Maxwell equations for the free the EMF. In the metrics

$$ds^2 = dt^2 - \sum_{i=1}^{3} A_i^2(t)\left(dx^i\right)^2, \tag{6}$$

they can be written as

$$\nabla_\mu F^{\mu\nu} = 0\,, \qquad \nabla_\mu (*F)^{\mu\nu} = 0\,, \tag{7}$$

$F_{\mu\nu}$ is electro-magnetic-field tensor and $(*F)^{\mu\nu}$ is adjoint magnitude, defined by the relation $(*F)^{\alpha\beta} = \frac{1}{\sqrt{-g}}[\alpha\beta\gamma\eta]F_{\gamma\eta}$, and $[\alpha, \beta, \gamma, \eta]$ is completely antisymmetric tensor [0123]=1.

The solutions of these equations can be represented in the form of electric- and magnetic-field vectors [1]

$$\mathbf{E}(t,\mathbf{x}) = \int d^3k e^{i\mathbf{kx}} \left[\mathcal{E}^\theta(t,\mathbf{k})\mathbf{e}_\theta(t,\mathbf{k}) + \mathcal{E}^\varphi(t,\mathbf{k})\mathbf{e}_\varphi(t,\mathbf{k})\right],$$

$$\mathbf{H}(t,\mathbf{x}) = \int d^3k e^{i\mathbf{kx}} \left[\mathcal{H}^\theta(t,\mathbf{k})\mathbf{e}_\theta(t,\mathbf{k}) + \mathcal{H}^\varphi(t,\mathbf{k})\mathbf{e}_\varphi(t,\mathbf{k})\right],$$

where

$$\mathbf{e}_\theta = \cos\theta_t \, \cos\varphi_t \frac{\mathbf{e}_1}{A_1} + \cos\theta_t \, \sin\varphi_t \frac{\mathbf{e}_2}{A_2} - \sin\theta_t \frac{\mathbf{e}_3}{A_3}\,,$$

$$\mathbf{e}_\varphi = -\sin\varphi_t \frac{\mathbf{e}_1}{A_1} + \cos\varphi_t \frac{\mathbf{e}_2}{A_2},$$

are the orthogonal vectors forming together with

$$\mathbf{e}_k = \sin\theta_t \, \cos\varphi_t \frac{\mathbf{e}_1}{A_1} + \sin\theta_t \, \sin\varphi_t \frac{\mathbf{e}_2}{A_2} + \cos\theta_t \frac{\mathbf{e}_3}{A_3}$$

the tetrad unit basis in the momentum space. The angles θ_t and φ_t are related with the spherical coordinates in the momentum space introduced via the relation

$$(k_1, k_2, k_3) = k\left(\sin\theta \, \cos\varphi, \, \sin\theta \, \sin\varphi, \, \cos\theta\right)\,,$$

using the formula

$$(\sin\theta_t \, \cos\varphi_t, \; \sin\theta_t \, \sin\varphi_t, \; \cos\theta_t) = \\ = \mu^{-1}\left(\frac{\sin\theta \, \cos\varphi}{A_1}, \, \frac{\sin\theta \, \sin\varphi}{A_2}\,, \, \frac{\cos\theta}{A_3}\right),$$

where

$$\mu^2 = \frac{\sin^2\theta \, \cos^2\varphi}{A_1^2} + \frac{\sin^2\theta \, \sin^2\varphi}{A_2^2} + \frac{\cos^2\theta}{A_3^2}\,.$$

The components $\mathcal{E}^\theta$, $\mathcal{E}^\varphi$, $\mathcal{H}^\vartheta$, $\mathcal{H}^\varphi$ can also be written as follows [2]:

$$
\begin{aligned}
\mathcal{E}^\theta(t,\mathbf{k}) &= \frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}}(y^+ + y^-)\,,\\
\mathcal{E}^\varphi(t,\mathbf{k}) &= \frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}k}\,\frac{d}{dt}(y^+ - y^-)\,,\\
\mathcal{H}^\theta(t,\mathbf{k}) &= \frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}}(y^+ - y^-)\,,\\
\mathcal{H}^\varphi(t,\mathbf{k}) &= -\frac{1}{\sqrt{2}(2\pi)^{3/2}(-g)^{1/4}}\,\frac{\mu}{b^{1/2}k}\,\frac{d}{dt}(y^+ + y^-)\,,
\end{aligned}
\tag{8}
$$

where

$$
b = \frac{1}{\sqrt{-g}}\left(A_2^2\cos^2\varphi + A_1^2\sin^2\varphi\right)\,,
$$

and the functions $y^\pm = y^r$ satisfy the equation

$$
\ddot{y}^r - \frac{\dot{b}}{b}\dot{y}^r + \left[k^2\mu^2 + rk\Delta\right]y^r = 0\,, \tag{9}
$$

$$
\Delta = b\frac{d}{dt}(a/b)\,;\quad a = \frac{\cos\theta\sin 2\varphi}{2\sqrt{-g}}\left(A_2^2 - A_1^2\right)\,.
$$

1.3. Quantum generation of photons in matter-antimatter annihilation

Assuming that at the time moment $t_{\rm in}$ on the background of the initially homogeneous and isotropic gravitational field in the Universe with Friedman metrics there arises the homogeneous anisotropic perturbation in accordance with matter-antimatter annihilation so that as a consequence the metrics can be represented as in Eq. (6). Let us assume also that at $t < t_{\rm in}$ the state of the EMF can be described with the density matrix with nonzero occupation number of the photons in the mode $n_0(\nu_0)$ corresponding to the black-body radiation. The latter is strictly constant at $t < t_{\rm in}$ and constant in the zeroth in respect to the anisotropy parameters approximation at $t > t_{\rm in}$:

$$
\frac{\partial}{\partial t}n_0(\nu_0) = 0.
$$

The frequency ν_0 is considered to be independent of time and equal to the radiation frequency in the current epoch. With the frequency at any time moment t it is related as follows

$$
\nu_0 A(t_0) = \nu(t)A(t), \tag{10}
$$

where $A(t)$ is the scale factor in the Friedman model at $t < t_{\text{in}}$ and $A^3 = (A_1 A_2 A_3)$ at $t > t_{\text{in}}$.

The external gravitational field of the anisotropic Universe brings about the increase of the photon number [2,4]. The particle number in the mode at the time moment $t > t_{\text{in}}$ satisfies the relation [2]

$$n\left(t, \nu_0, \theta, \varphi\right) = n_0(\nu_0) + n_1\left(t, \nu_0, \theta, \varphi\right) + n_q\left(t, \nu_0, \theta, \varphi\right). \tag{11}$$

Here

$$n_1\left(t, \nu_0, \theta, \varphi\right) = n_0(\nu_0)\delta\left(t, \nu_0, \theta, \varphi\right), \tag{12}$$

where $|\delta| \ll 1$ is the correction, describing the anisotropic distribution over momenta which arises due to the anisotropic expansion of the photons already existing to the time moment t_{in}. As to the quantity

$$n_q\left(t, \nu_0, \theta, \varphi\right) = 2\left(\sum_{r-\pm 1} |\beta^r\left(t, \nu_0, \theta, \varphi\right)|^2\right)\left(2n_0(\nu_0) + 1\right), \tag{13}$$

it is the additional number of photons which arose due to their generation by matter-antimatter annihilation in the non-stationary gravitational field. β^r is the coefficient of the Bogoliubov transformation at the transition to the time-independent operators of the generation-annihilation of photons bringing to the diagonal form the instantaneous Hamiltonian of the quantised the EMF at the time moment t on the operators of the generation-annihilation, in terms of which the Hamiltonian has the diagonal form at the initial time moment t_{in} ; $|\beta^r(t, \nu_0, \theta, \varphi)|^2$ is the density of the probability of the generation of a photon with a certain frequency ν_0, direction of the wave vector θ, φ, and the spin projection r on the direction of the wave vector.

Let us generalise the relations (11) – (13) to the case when the matter of interest is the particle number in the mode, which polarisation vector is oriented along a certain direction in the coordinate frame connected with the wave vector of the photon. Then by analogy to Eq. (4) we can introduce the symbolic vector [2]

$$\mathbf{n}\left(t, \nu_0, \theta, \varphi\right) = \frac{1}{2}\,\frac{c^3}{h\nu^3(t)}\begin{pmatrix} I\left(t, \nu_0, \theta, \varphi\right) + Q\left(t, \nu_0, \theta, \varphi\right) \\ I\left(t, \nu_0, \theta, \varphi\right) - Q\left(t, \nu_0, \theta, \varphi\right) \\ U\left(t, \nu_0, \theta, \varphi\right) \end{pmatrix}, \tag{14}$$

where I, Q, U are the Stokes parameters of the EM radiation. By analogy with Eq. (11), $\mathbf{n}$ can be represented as

$$\mathbf{n}\left(t, \nu_0, \theta, \varphi\right) = \mathbf{n}_0(\nu_0) + \mathbf{n}_1\left(t, \nu_0, \theta, \varphi\right) + \mathbf{n}_q\left(t, \nu_0, \theta, \varphi\right), \tag{15}$$

where

$$\mathbf{n}_0(\nu_0) = n_0(\nu_0)\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix},$$

which corresponds to the isotropic nonpolarised radiation. In the case when there is no scattering of the photons on the electrons of the cosmic plasma, $\mathbf{n}_1$ can be represented as [2]

$$\mathbf{n}_1\left(t,\nu_0,\theta,\varphi\right) = n_0(\nu_0)\left(\alpha(t,\nu_0)\mathbf{a}(\theta,\varphi) + \bar{\alpha}(t,\nu_0)\bar{\mathbf{a}}(\theta,\varphi)\right),$$
$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\left(\cos^2\theta - \frac{1}{3}\right), \bar{\mathbf{a}} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\left(1-\cos^2\theta\right)\cos 2\varphi. \tag{16}$$

This corresponds to the start of the dependence on angles, i.e. anisotropy, in the distribution of the photons over momenta. The coefficients α and $\bar{\alpha}$ characterise the degree of the quadrupole anisotropy of the CMB radiation.

The quantity $\mathbf{n}_q$ describes the contribution of the quantum effects in matter-antimatter annihilation process. In the linear with respect to the anisotropy parameters approximation of the metrics (6) it can be represented in the form:

$$\mathbf{n}_q\left(t,\nu_0,\theta,\varphi\right) = \frac{2n\alpha(\nu_0)+1}{2}\,\frac{c^3}{h\nu^3(t)}\begin{pmatrix} Q^{\rm ann}\left(t,\nu_0,\theta,\varphi\right) \\ -Q^{\rm ann}\left(t,\nu_0,\theta,\varphi\right) \\ 2U^{\rm ann}\left(t,\nu_0,\theta,\varphi\right) \end{pmatrix}. \tag{17}$$

The quantities $I^{\rm ann}$, $Q^{\rm ann}$, $U^{\rm ann}$ are the annihilation Stokes parameters (ASP), evaluated for the case when the initial state of the EMF at the time of the matter-antimatter annihilation $t_{\rm in}$ was only the CMB radiation.

2. Polarization of Photons due to the quantum effects in matter-antimatter Universe

In general (i.e. not in the linear approximation) the Stokes parameters are related with the polarisation density matrix of the quantum effects of the EMF in matter-antimatter Universe, introduced in [1] as a new characteristics of the latter, as usually (see [2]):

$$J^{ab} = \frac{1}{2}\begin{pmatrix} I^{\rm ann}+Q^{\rm ann} & U^{\rm ann}-iV^{\rm ann} \\ U^{\rm ann}+iV^{\rm ann} & I^{\rm ann}-Q^{\rm ann} \end{pmatrix}, \tag{18}$$

where

$$J^{ab\,ann} = J_{+}^{ab\,ann} + J_{-}^{ab\,ann}, \quad J_{\pm}^{ab\,ann}(t,\mathbf{k}) = \frac{1}{2}\,\frac{hk}{\mu(t,\theta,\varphi)} \times$$
$$\times \langle 0_{t_{\rm in}}|N_t\left[\hat{\mathcal{E}}^a(t,\mathbf{x},\mathbf{k}),\hat{\mathcal{E}}^b(t,\mathbf{x},\mathbf{k})\right]|0_{t_{\rm in}}\rangle.$$

Here $\hat{\mathcal{E}}^a(t,\mathbf{x},\mathbf{k})$ are the components of the spectral component of the vector of electric field (8), multiplied by $\exp(i\mathbf{kx})$.

The calculations, carried out in [2], have shown that $J^{ab\,ann} = 0$, i.e. according to the common interpretation [3], the generating photons do not have an admixture of the circular polarisation.

The symmetric part of the polarisation tensor could be expressed via spectral components of the averages of the operator of energy-momentum tensor

$$T^{\text{ann}}_{\mu\nu}(t) = \int d\varphi d\theta \sin\theta \int dK_0\left(t,k,\theta,\varphi\right)\ \widetilde{T}^{\text{ann}}_{\mu\nu}\left(t,k,\theta,\varphi\right),$$
$$K_0\left(t,k,\theta,\varphi\right) = ck\mu\left(t,\theta,\varphi\right)\ ,$$

as follows

$$J^{ab}_{+} = G^{ab}_{\mu\nu}\widetilde{T}^{\text{ann}}_{\mu\nu}.$$

The specific form of the components $G^{ab}_{\mu\nu}$ has been evaluated in [5]. Also in [2,5] is given the explicit form of the spectral components $\tilde{T}^{\text{ann}}_{\mu\nu}$. So the ASP can be presented as follows [2] :

$$\left(I^{\text{ann}}, Q^{\text{ann}}, U^{\text{ann}}, V^{\text{ann}}\right) = \frac{hk^3}{V}\sum_{r=\pm1}\left(2s^r, u^r, r\tau^r, 0\right), \tag{19}$$

where the functions s^r, u^r, τ^r satisfy the set of the equations [2]

$$\begin{cases} \dot{s}^r = \frac{W}{2}u^r + r\frac{\overline{W}}{2}\tau^r, \\ \dot{u}^r = W\left(2s^r+1\right) - \left(r\overline{W} + 2cK_0\right)\tau^r, \\ \dot{\tau}^r = r\overline{W}\left(2s^r+1\right) + \left(r\overline{W} + 2cK_0\right)u^r, \end{cases} \tag{20}$$
$$\left(u^r\right)^2 + \left(\tau^r\right)^2 = 4s^r\left(s^r+1\right)$$

with the initial values $s^r(t_{\text{in}}) = u^r(t_{\text{in}}) = \tau^r(t_{\text{in}}) = 0$. The quantities W and $\overline{W}$ in the metrics, linear in respect to the anisotropy parameters of metrics (6) are as follows:

$$\begin{aligned} W &= \left(1-\cos^2\theta\right)\Delta H + \frac{\overline{\Delta H}}{2}\left(1+\cos^2\theta\right)\cos 2\varphi, \\ \overline{W} &= -\cos\theta\,\sin 2\varphi\overline{\Delta H}, \end{aligned} \tag{21}$$

where

$$\Delta H = = H - \frac{1}{2}\left(H_1 + H_2\right), \quad \overline{\Delta H} = H_1 - H_2,$$
$$H^3 = (H_1 H_2 H_3), \quad H_i = \dot{A}_i / A_i$$

(the parameters ΔA, $\overline{\Delta A}$, A could be introduced similarly).

The set of the equations (20) plays the part of the transfer equations for ASP. Let us analyse the expressions (19), solving (20) via expansion of the functions in a power series in respect to small parameter $\tilde{h}$ which is introduced as

$$\Delta H \to \tilde{h}\Delta H, \quad \overline{\Delta H} \to \tilde{h}\overline{\Delta H}, \Delta A \to \tilde{h}\Delta A, \quad \overline{\Delta A} \to \tilde{h}\overline{\Delta A}.$$

In doing so,

$$s^r = \sum_{n=0} \tilde{h}^n s_n^r\,, \quad u^r = \sum_{n=0} \tilde{h}^n u_n^r\,, \quad \tau^r = \sum_{n=0} \tilde{h}^n \tau_n^r\,. \tag{22}$$

The expansions of W and $\overline{W}$ are given with (21). Further we shall keep in the expansion (22) only linear terms in respect to $\tilde{h}$. In the zeroth approximation in respect to $\tilde{h}$ it follows from (20), (21), taking into the account the initial values, that

$$s_0^r = u_0^r = \tau_0^r = 0.$$

This means that there are no photon quantum effects in isotropic case. In the linear approximation the set of the equations for s_1^r, u_1^r, τ_1^r is

$$\dot{s}_1^r = 0, \quad \dot{u}_1^r = W - 2\nu\tau_1^r, \quad \dot{\tau}_1^r = r\overline{W} + 2\nu u_1^r$$

(in the zeroth approximation in respect to $\tilde{h}$ $K_0(t,k,\theta,\varphi) = k_0/A(t) \equiv \nu(t)$). Solving this set, one can obtain the expressions for ASP in the linear approximation:

$$(I^{\text{ann}},\, Q^{\text{ann}},\, U^{\text{ann}}) = \tag{23}$$

$$= \frac{2h\nu^3}{c^3} \int\limits_{t_{\text{in}}}^{t} \Big(0, W(t'), \overline{W}(t')\Big) \cos\left(2(\Omega(t) - \Omega(t'))\right) dt',$$

$$\Omega(t) = \int dt\nu(t).$$

Such a distribution of the ASP quantum effects in the anisotropic gravitational field is unusual from the viewpoint of the classical electrodynamics [6]. The reason is the structure of the vacuum energy-momentum tensor of the EMF in the external gravitational field. *Zeldovich* and *Starobinsky* [7] have remarked that quantum effects of the material field in the external anisotropic gravitational field bring about the breaking of the condition of the energy dominance of the energy-momentum tensor (EMT) of these fields.) This fact shows itself in different dependence of the EMT components on the anisotropy parameters of the metrics (6), namely:

$$T_0^0 \sim \tilde{h}^2, \qquad T^{ik} \sim \tilde{h}, \qquad i,k = 1,2,3.$$

The fact that EMT does not satisfy the condition of the energy dominance means that it contains both the contribution from the really generated particles and the contribution of the EMF in matter-antimatter annihilation process. It is impossible to divide the energy-momentum tensor into the aforementioned contributions as it has been indicated in [2].

Let us bring $\mathbf{n}_q$ to the form analogous that of $\mathbf{n}_1$, singling out explicitly the dependence on the angles θ and φ. To this end we use (21) and (23), then

$$\begin{aligned}\mathbf{n}_q(t,\nu_0,\theta,\varphi) &= \frac{2n_0(\nu_0)+1}{2} \times \\ &\times \left(\beta_q(t,\nu_0)\mathbf{b}(\theta,\varphi)+\bar{\beta}_q(t,\nu_0)\bar{\mathbf{b}}(\theta,\varphi)\right),\end{aligned} \tag{24}$$

here

$$\beta_q(t) = \int\limits_{t_{\rm in}}^{t} \Delta H(t')\cos\left(2(\Omega(t)-\Omega(t'))\right)dt'. \tag{25}$$

The expression for $\bar{\beta}_q$ can be obtained from (25), substituting $\overline{\Delta H}$ for ΔH. Vectors $\mathbf{b}$ and $\bar{\mathbf{b}}$ are defined via the relations (analogously as was defined in [8] by Basco and Polnarev):

$$\mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}(1-\cos^2\theta), \quad \bar{\mathbf{b}} = \frac{1}{2}\begin{pmatrix} (1+\cos^2\theta)\cos 2\varphi \\ -(1+\cos^2\theta)\cos 2\varphi \\ 4\cos\theta\,\sin 2\varphi \end{pmatrix}.$$

Assuming that $n_0 \gg 1$ and substituting (16), (24) into (15), we obtain

$$\mathbf{n} = \mathbf{n}_0 + n_0\left[\alpha\mathbf{a} + \beta_q\mathbf{b} + \bar{\alpha}\bar{\mathbf{a}} + \bar{\beta}_q\bar{\mathbf{b}}\right]. \tag{26}$$

The quantities β_q and $\bar{\beta}_q$ are related with the degree of the linear polarisation of the EMF in matter-antimatter annihilation. The relation (26) is similar to that derived in [8] under the solution the radiation transfer equation taking into the account Thomson scattering of the photons on the electrons of the cosmic plasma.

The quantities which undergo the measurement in the course of experiment are the Stokes parameters I, Q and U. Let us evaluate, for example, the Stokes parameter Q in the Heckman-Schüking model [2]. According to Eqs. (14) and (26) we have

$$Q(t,\nu_0,\theta,\varphi) = \frac{2h\nu_0^3}{c^3}n_0(\nu_0)\left(1-\cos^2\theta\right)\beta_q(t,\nu_0).$$

The dependence of Q on the time t is determined with the quantity β_q (25). Let us transform it to more clear form. To this end let us temporarily change in the integral (25) the integration variable X as follows:

$$\begin{aligned} X &= (1+\chi)^{-1/2}, \quad \Delta H = \Delta H_0(1+\chi)^3 = \Delta H_0/X^6, \\ dt &= -\frac{d\chi}{H_0(1+\chi)^{5/2}} = \frac{2}{H_0}X^2 dX, \nu(t) = \nu_0(1+\chi) = \nu_0 X^{-2} \end{aligned}$$

where ΔH_0, H_0, ν_0 are the current values of the corresponding parameters. By this change of variable the integral is brought to the form

$$\beta_q = \frac{\Delta H_0}{H_0} \int\limits_{X_{\rm in}}^{X} \frac{\cos\lambda(X - X')}{X'^4} dX', \tag{27}$$

where $\lambda = 4\nu_0/H_0 \approx 10^{30}$ is a large parameter. Estimating (27) asymptotically in λ, we obtain

$$\beta_q = \frac{\Delta H_0}{H_0} \frac{1}{\lambda X_{\rm in}^4} \sin\left(\lambda(X - X_{\rm in})\right). \tag{28}$$

Let us change in the last formula from the current variable X to the synchronous time t according to the relation $X = (3H_0 t/2)^{1/3}$. The time t is synchronous cosmological time, counted from the singularity. Let us divide it into two summands as follows:

$$t = t_0 + t', \qquad t' \ll t_0,$$

where $t_0 = \frac{2}{3H_0}$ is the time counted from the beginning of the expansion, corresponding to the current epoch, and t' is the current time, for example, the period, during which the observations take place. Then

$$\begin{aligned} Q(t, \nu_0, \theta, \varphi) \; &= \; \frac{2h\nu_0^3}{c^3} n_0(\nu_0)(1 - \cos^2\theta) \times \\ &\times \; \frac{\Delta H_0}{H_0} \frac{\chi_{\rm in}}{\lambda} \sin\left(2\nu_0 t' + \lambda(1 - X_{\rm in})\right). \end{aligned}$$

Let us discuss the possibility of experimental measuring of Q. The power of the polarised component of interest for us of the EMF radiation, hitting the aerial of the radio-telescope, with the directivity diagram $P_n(\theta, \varphi)$ and the effective area of the surface $A_{\rm eff}$ per unit frequency band is defined as follows [9]:

$$W(t) = \frac{1}{2} A_{\rm eff} \left| \int\int\limits_{\Omega} Q(t, \nu_0, \theta, \varphi) P_n(\theta, \varphi) d\Omega \right|. \tag{29}$$

Separating the dependence on angle, we bring (29) to the form

$$W(t) = \frac{2h\nu_0^3}{c^3} n_0(\nu_0) \Omega_{\rm eff}^{Q} \left| \beta_q(t, \nu_0) \right|.$$

The quantity $W(t)$ is the instantaneous power of the signal in the aerial. On the load of the aerial the voltage is induced, the square of which is proportional to the instantaneous power, so that the current at the output of the detector is as follows [9]:

$$i^{Q}_{\rm det}(t) = k'\hat{U}^2(t) = k'W(t) = k'\Omega^{Q}_{\rm eff}\,\frac{2h\nu_0^3}{c^3}n_0(\nu_0)\left|\beta_q(t,\nu_0)\right| .$$

After averaging over time, we obtain

$$\overline{i^{Q}_{\rm det}} = k'\left(\Omega^{Q}_{\rm eff}\right)\left(\frac{2h\nu_0^3}{c^3}n_0(\nu_0)\right)\frac{\Delta H_0}{H_0}\,\frac{\chi^2_{\rm in}}{\lambda}\,\frac{2}{\pi} .$$

Similarly, measuring the Stokes parameter I in the zeroth approximation with respect to the anisotropy parameter ΔH, we arrive at

$$\overline{i^{I}_{\rm det}} = k'\left(\Omega^{T}_{\rm eff}\right)\left(\frac{2h\nu_0^3}{c^3}n_0(\nu_0)\right) .$$

The observable degree of polarisation turns out to be

$$P = \frac{\overline{i^{Q}_{\rm det}}}{\overline{i^{I}_{\rm det}}} = \frac{2}{\pi}\,\frac{\Omega^{Q}_{\rm eff}}{\Omega^{I}_{\rm eff}}\,\frac{\Delta H_0}{H_0}\,\frac{\chi^2_{\rm in}}{\lambda} . \qquad (30)$$

Thus, the degree of polarisation of the EMF due to the quantum effects in matter-antimatter process turns out to be of essence only in the case, when the anisotropy of the metrics has manifested itself sufficiently early.

The background result of this paper constitutes the idea, which attempts to explain some linear polarization of the EMF in the Universe as produced by quantum effects in matter-antimatter annihilation process. Thus, it turns out that in the case of the transparent medium when there is no scattering of the photons, the EMF radiation in the homogeneous anisotropic and non-stationary Universe becomes linearly polarised due to the quantum effects of the photon generation by matter-antimatter annihilation. It is remarkable that the angular dependence of the photon number is quadrupole and completely coincides with that arising under the scattering of the CMB photons on electrons in the epoch of the recombination or the secondary ionization [1–4]. On the other hand, the generation of the photons in matter-antimatter annihilation process and the polarization of the EMF take place in the external anisotropic gravitational field in the space of the I type according to Bianchi [1–4].

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