Orbits of S2 star in Yukawa gravity: simulations vs observations *

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Abstract

We investigate some possible observational signatures of Yukawa gravity at Galactic scales. We performed simulations in Yukawa gravity potential and analyzed the obtained trajectories of S2 star around Galactic Centre. We compare the obtained theoretical results for S2 star orbit in the Yukawa potential with two independent sets of observations of the S2 star obtained by New Technology Telescope/Very Large Telescope (NTT/VLT), as well as by Keck telescope. Our results show that the most probable value for the parameter Λ in Yukawa gravity potential in the case of S2 star is around 5000 - 7000 AU. At the same time, we were not able to obtain reliable constrains on the universal constant δ of Yukawa gravity, because the current observations of S2 star indicated that it may be highly correlated with parameter Λ. We obtained that the Yukawa gravity potential induces precession of S2 star orbit in the same direction as General Relativity for $\delta > 0$ and for $\delta < -1$, and in the opposite direction for $-1 < \delta < 0$.

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1. Introduction

Extended Theories of Gravity [1] have been proposed like alternative approaches to Newtonian gravity in order to explain galactic and extragalactic dynamics without introducing dark matter [2, 3]. $f(R)$ gravity is a type of modified gravity which generalizes Einstein's General Relativity and it was first proposed in 1970 by Buchdahl [4]. $f(R)$ gravity presented a family of models, each one defined by a different function of the Ricci scalar. The simplest case is just the General Relativity. In the case of $f(R)$ gravity, one assumes a generic function f of the Ricci scalar R (in particular, analytic functions) and searches for a theory of gravity having suitable behavior at different scales: for short distances, Solar system, spiral galaxies and galaxy clusters.

Yukawa-like corrections have been obtained in the framework of $f(R)$ gravity as a general feature of these theories [5, 6, 7, 8]. The physical meaning of such corrections needs to be confirmed at small and large scale lengths. Different experimental, geophysical and astronomical constraints on Yukawa violations of the gravitational inverse square law are given in Figs. 9 and 10 from [9] for different ranges. From these results we can conclude that the Yukawa term is relatively well constrained for the short ranges. Also, for longer distances Yukawa corrections have been successfully applied to clusters of galaxies [10, 11, 7]. Further tests are needed in order to set good constraints on Yukawa corrections, especially in the ranges of Galactic stellar dynamics.

S-stars are young stars that closely orbit the massive compact object at the center of Milky Way, named Sgr A* [12, 13, 14, 15]. These stars, together with recently discovered dense gas cloud falling towards the Galactic Centre [15], indicate that the massive central object is a probably black hole. We find astrometric observations for one of them, called S2. These observation indicate that its orbit maybe do not yield closed ellipses [12, 16].

2. Theory

A general gravitational potential, with a Yukawa correction, can be obtained in the Newtonian limit of any analytic $f(R)$-gravity model.

$$\mathcal{S} = \int d^4x\sqrt{-g}\,[f(R) + \mathcal{X}\mathcal{L}_m]\,. \tag{1}$$

The simplest extension of General Relativity is achieved assuming:

$$R \rightarrow f(R)\,, \qquad \omega(\phi) = 0 \tag{2}$$

One can assume analytic Taylor expandable $f(R)$ functions with respect to the value $R = 0$ that is the Minkowskian background [17]:

$$f(R) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} R^n = f_0 + f_1 R + \frac{f_2}{2} R^2 + \ldots \tag{3}$$

At the order $\mathcal{O}(0)$, the field equations give the condition $f_0 = 0$ and then the solutions at further orders do not depend on this parameter. Considering the first term in R, f_0 has the meaning of a cosmological constant.

In $f(R)$-gravity, the scalar curvature R of the Hilbert - Einstein action, is replaced by a generic function $f(R)$. As a result, in the weak field limit [18], the gravitational potential is found to be Yukawa-like [19, 20]:

$$\Phi\,(r) = -\frac{GM}{(1+\delta)r}\left[1 + \delta e^{-\left(\frac{r}{\Lambda}\right)}\right], \tag{4}$$

where $\Lambda^2 = -f_1/f_2$ is an arbitrary parameter (usually referred to as the range of interaction), depending on the typical scale of the system under consideration and $\delta = f_1 - 1$ is a universal constant.

In order to constrain the parameters, we simulate orbits of S2 star in Yukawa-like gravity potentials and fit them to the astrometric observations obtained by New Technology Telescope/Very Large Telescope (NTT/VLT)[12].

The simulated orbits of S2 star were obtained by numerical integration of differential equations of motion in Yukawa gravitational potential:

$$\dot{\mathbf{r}} = \mathbf{v}, \quad \mu\ddot{\mathbf{r}} = -\nabla\Phi\left(\mathbf{r}\right), \tag{5}$$

where μ is so called reduced mass in the two-body problem. In our calculations we assumed that the mass of central black hole is $M_{BH} = 4.3 \times 10^6 M_\odot$ and that the distance to the S2 star is $d_\star = 8.3$ kpc [12]. Detailed description of calculation and fitting procedure is given in the papers [21, 22, 23]. Perturbations from other members of the stellar cluster, as well as from some possibly existing extended structures composed from visible or dark matter [24, 25] were neglected due to simplicity reasons.

3. Results and discussions

Our goal is that the Yukawa-like correction, coming from $f(R)$ gravity, can be used in order to fix the coefficients in the expansion (3). For the expansion up to the second order, we have 2 parameters to fix. According to [2], the relations between f_1, f_2 and δ and Λ parameters are $f_1 = 1 + \delta$, $f_2 = -(1 + \delta)/(\Lambda^2)$. We have to find the minimal values of the reduced χ^2 in order to determine f_1 and f_2 assuming $f_0 = 0$. This allows to reconstruct $f(R)$ models up to the second order.

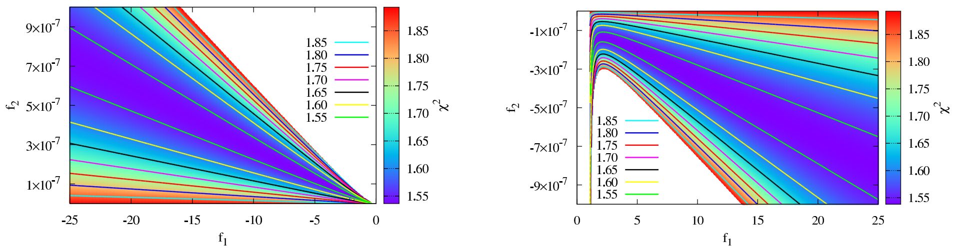

Figure 1: The maps of reduced χ^2 over the $\{f_1 - f_2\}$ parameter space of $f(R)$ gravity in case of NTT/VLT observations of S2 star which give at least the same ($\chi^2 = 1.89$) or better fits ($\chi^2 < 1.89$) than the Keplerian orbits. The left panel corresponds to f_1 in $[-25, 0]$, and the right panel to f_1 in $[0, 25]$. A few contours are presented for specific values of reduced χ^2 given in figure's legend.

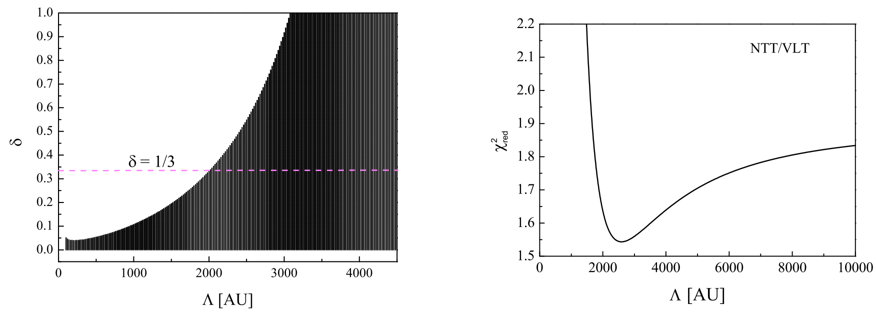

Figure 2: (Left panel) The parameter space for Yukawa gravity under the constraint that, during one orbital period, S2-like star orbits in Yukawa gravity differ less than ϵ from the corresponding orbits in Newtonian gravity for the value of parameter $\epsilon = 0.01$. (Right panel) The reduced χ^2 for δ=1/3 as a function of Λ in case of NTT/VLT observations.

Fig. 1 presents the maps of the reduced χ^2 over the $\{f_1 - f_2\}$ parameter space for all simulated orbits of S2 star which give at least the same or better fits to the NTT/VLT observations of S2 star than the Keplerian orbits ($\chi^2 = 1.89$). In a large region of the parameter space, χ^2 of the orbits in modified potential is less than the value in Newtonian potential. It comes that we cannot constrain both f_1 and f_2 using only the observed S2 orbits because these two parameters are strongly correlated. We can con-

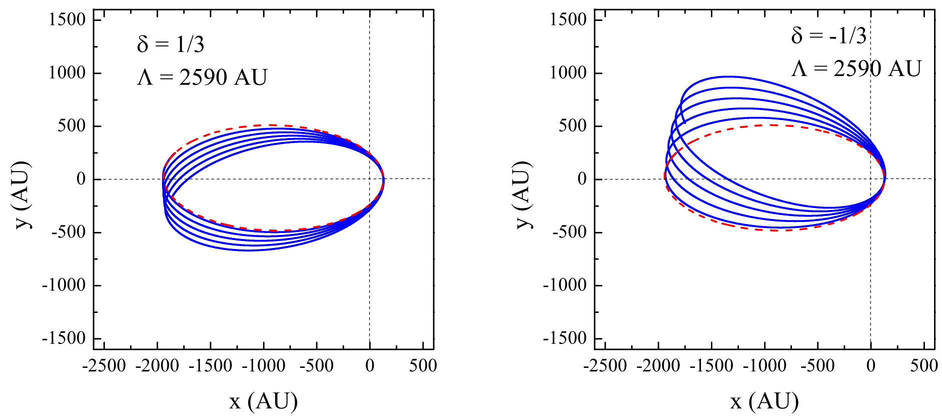

Figure 3: Comparisons between the orbit of S2 star in Newtonian gravity (red dashed line) and Yukawa gravity (blue solid line) during 5 orbital periods for $\Lambda = 2.59 \times 10^3$ AU. In the left panel $\delta = +1/3$, and in the right $\delta = -1/3$, respectively.

strain only their ratio f_1/f_2.

In Fig. 2 (left panel) we presented the parameter space for Yukawa gravity under the constraint that, during one orbital period, S2-like star orbits in Yukawa gravity differ less than ϵ from the corresponding orbits in Newtonian gravity, for the value of parameter ϵ=0.01. The value of parameter ϵ has very strong influence of constraints on the parameters of Yukawa gravity theories. Additionally, in Fig. 2 we designated value of $\delta = 1/3$ by line.

Fig. 2 (right panel) presents the reduced χ^2 for all fits with fixed value of δ=1/3 as a function of Λ which was varied from 10 to 10 000 AU. In the case of NTT/VLT observations the minimum of reduced χ^2 is 1.54 and is obtained for $\Lambda = 2.59 \times 10^3$ AU, while in the case of NTT/VLT+Keck combined data set the minimal value of 3.24 is obtained for $\Lambda = 3.03 \times 10^3$ AU. For both cases the reduced χ^2 for Keplerian orbits ($\delta = 0$) are 1.89 and 3.53, respectively. These values are significantly higher than the corresponding minima for $\delta = 1/3$. We can conclude that Yukawa gravity describes observed data even better than

Newtonian gravity and that $\delta = 1/3$ is valid value at galactic scales.

The simulated orbits of S2 star around the central object in Yukawa gravity (blue solid line) and in Newtonian gravity (red dashed line) for $\Lambda = 2.59 \times 10^3$ AU and $\delta = +1/3$ (left panel) and $\delta = -1/3$ (right panel) during 5 periods, are presented in Fig. 3. We can notice that for $\delta = -1/3$ the precession has the negative direction and when $\delta = +1/3$ the precession has the positive direction (like in General Relativity). Our analysis shows that the Yukawa gravity potential induces precession of S2 star orbit in the same direction as General Relativity for $\delta > 0$ and for $\delta < -1$, and in the opposite direction for $-1 < \delta < 0$ as in the case of extended mass distribution or in R^n gravity [21].

4. Conclusions

In this paper we constrained the parameters of Yukawa gravity using the observed positions of S2 orbit around the Galactic Centre. Our investigation shows that:

1. for vanishing δ, we recover the Keplerian orbit of S2 star;
2. the most probable value for Yukawa gravity parameter Λ in the case of S2 star, is around 5000 - 7000 AU;
3. the two parameters of Yukawa gravity Λ and δ are highly correlated in the range ($0 < \delta < 1$). For $\delta > 2$ they are not correlated;
4. if we take $\delta = +1/3$, i.e. the same universal constant δ which was successfully applied to clusters of galaxies [10, 11] and rotation curves of spiral galaxies [7], we also get a good agreement (better than in the Keplerian case) in the case of observations of S2 star orbit;

5. the scale parameter of Yukawa gravity in the case of S2 star for $\delta = +1/3$ is about: $\Lambda \approx 3000 \pm 1500$ AU;
6. for $\delta = +1/3$ there is orbital precession in positive direction like in General Relativity, and for $\delta = -1/3$ the precession has negative direction, as in the case of extended mass distribution or in R^n gravity [21];
7. the connection between coefficients f_1 and f_2 and Yukawa gravity parameters Λ and δ are following: $\Lambda^2 = -f_1/f_2$ $\delta = f_1 - 1$. We cannot constrain both f_1 and f_2 using only the observed S2 orbits because these two parameters are strongly correlated. We can constrain only their ratio f_1/f_2.

The newest astrometric data for the star S2 of NTT/VLT measurements and Keck measurements indicate that maybe the S2 orbits deviate from the Keplerian case, due to the relativistic precession. The accuracy of the observations is constantly improving from around 10 mas during the first part of the observational period, currently reaching less than 1 mas. With that currently limit one can not say for sure that S2 star orbit really deviates from the Newtonian case. We hope that future observations with advanced facilities, such as GRAVITY[26] or METIS[27], will be able to verify these claims.

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